Tutorial Assignment 2: Ideal Gas and Electrons

Write clearly. Answer to 4 significant figures. Marks given for units and explanation of calculations.

1. A box has the shape of a cube with side a. It contains 1 mole of neon gas (relative atomic mass = 20) at 250 K. The box has one corner at the origin and 3 of its edges along the positive x, y and z axes. It has zero potential inside and infinite potential at the walls.

i) (k_x, k_y, k_z) can be represented by a point in k space. State the quantisation conditions and find the smallest distance between points. [2]

ii) If we divide the volume in k space into identical cubes centred at all points of (i), explain what would be the volume of each cube? [2]

iii) Find an approximate expression for a spherical shell of radius k and thickness dk. Why does only one eighth of this shell contain states of a particle. [2]

iv) What is meant by a state of the particle in the ideal gas? What is density of states? [2]

v) How many states are there in the spherical shell in (iii)? Derive the density of states $g(k)$ in terms of volume V of the cube. [2]

vi) State the formula for energy ε of the particle in terms of k. Transform $g(k)$ to $g(\varepsilon)$. [2]

vii) What are the physical meanings of $g(\varepsilon)d\varepsilon$, $f(\varepsilon)g(\varepsilon)d\varepsilon$, and $\varepsilon f(\varepsilon)g(\varepsilon)d\varepsilon$? ($f(\varepsilon)$ is the average number of particles in a state.) [2]

viii) From (vi) and (vii), derive an integral expression for the total kinetic energy U of all particles. $f(\varepsilon)$ is the Boltzmann distribution. [2]

ix) Find the smallest spacing between wavevectors when a = 10 cm. Find the energy when the wavevector of a neon atom is equal to this smallest spacing. [2]

x) If the energy in (ix) is the average energy of the neon atoms, find the corresponding temperature. What would happen to the neon atoms at this temperature if they are bosons? Why would the ideal gas model fail? [2]

2. In silver metal, each atom provides one conduction electron.

i) Write down the formula for the energy distribution $f(\varepsilon)$ of the conduction electrons. What is the physical meaning of the number given by $f(\varepsilon)$? [2]

ii) Above 0 K, some electrons are excited. These mainly come from a small interval of energy below Fermi level. Estimate this interval, giving the answer in terms of T. **[2]**

iii) g(ε) is the ideal gas density of states and E_F is the Fermi energy. In terms of $g(E_F)$, derive an approximate expression for n, the number excited electrons at T. **[2]**

iv) The exited electrons are raised above Fermi level. Use (i) to derive an approximate distribution of the excited electrons. **[2]**

v) Show that the distribution of the excited electrons is a Boltzmann distribution. What does this result suggest about the excited electrons? **[2]**

vi) State the formula for average energy of a particle in an ideal gas at T. Using (iii), find the formula for total energy of the excited electrons. **[2]**

vii) Hence find a formula for the heat capacity due to the excited electrons. [2]

viii) Find the Fermi energy of the conduction electrons in silver. Molar volume of silver is 10.72 cm^3. **[2]**

ix) Write down the expression for density of states g(ε). Using (vii) and (viii), calculate the electronic heat capacity of one mole of silver at 1 K. [2]

x) If all of the conduction electrons of the silver behave as an ideal gas, what would be the heat capacity? Why is the answer in (ix) so much smaller than this? [2]